

Exercice 1

$$\begin{pmatrix} 2 & -2 \\ 6 & 4 \end{pmatrix} \times \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Leftrightarrow \begin{cases} 2a - 2c = 1 \\ 6a + 4c = 0 \\ 2b - 2d = 0 \\ 6b + 4d = 1 \end{cases} \Leftrightarrow \begin{cases} a = \frac{1}{5} \\ c = -\frac{3}{10} \\ b = \frac{1}{10} \\ d = \frac{1}{10} \end{cases}$$

$$A^{-1} = \begin{pmatrix} \frac{1}{5} & \frac{1}{10} \\ -\frac{3}{10} & \frac{1}{10} \end{pmatrix}$$

Exercice 2

$$a) \begin{cases} 3x + 2y - z = -1 \\ x - y + z = 3 \\ 2x - 4y + 5z = 2 \end{cases} \Leftrightarrow \begin{pmatrix} 3 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & -4 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$$

$$\Leftrightarrow A X = B \quad \text{avec} \quad A = \begin{pmatrix} 3 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & -4 & 5 \end{pmatrix} \quad B = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$\det A = -7 \neq 0$ donc A est inversible

$$\begin{aligned} A^{-1} A X &= A^{-1} B \\ \Leftrightarrow \underbrace{A^{-1} A}_I X &= A^{-1} B \end{aligned}$$

$$\Leftrightarrow X = A^{-1} B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & -4 & 5 \end{pmatrix}^{-1} \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 15/7 \\ -46/7 \\ -40/7 \end{pmatrix}$$

Exercice 3

$$\begin{cases} x + y + z = 122 \\ 10y + 7z = 580 \\ 5x + 2y + 3z = 440 \end{cases} \Leftrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 10 & 7 \\ 5 & 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 122 \\ 580 \\ 440 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 10 & 7 \\ 0 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 122 \\ 580 \\ 440 \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 52 \\ 30 \\ 40 \end{pmatrix}$$

La hotline a reçu 52 appels de type F, 30 appels de type C et 40 appels de type T.

Exercice 4

$$A^3 + aA^2 + bA + cI_3 = 0$$

$$\Leftrightarrow \begin{pmatrix} 1 & 0 & 4 \\ 4 & 1 & 12 \\ 4 & 4 & 1 \end{pmatrix} + \begin{pmatrix} a & a & 0 \\ 3a & 3a & a \\ a & 0 & 4a \end{pmatrix} + \begin{pmatrix} 0 & 0 & b \\ b & 0 & 3b \\ b & b & 0 \end{pmatrix} + \begin{pmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} 1 + a + c = 0 \\ a = 0 \\ 4 + b = 0 \end{cases} \Leftrightarrow \begin{cases} a = 0 \\ c = -1 \\ b = -4 \end{cases}$$

Finalement on a: $A^3 - 4A - I = 0$

$$\Leftrightarrow A(A^2 - 4I) = I$$

donc A est inversible avec $A^{-1} = A^2 - 4I = \begin{pmatrix} -3 & 1 & 0 \\ 3 & -1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$