Exercice 1

$$A = (1+2i)^3 = -11-2i$$

$$B = \frac{1}{(2+3i)^3} = \frac{-46-9i}{2197}$$

Exercice 2

$$\left.\begin{array}{l} z_{\vec{AB}} = z_B - z_A = (3+3i) - (-2+5i) = 5-2i \\ z_{\vec{AC}} = z_C - z_A = (13-i) - (-2+5i) = 15-6i \end{array}\right\}$$

on obtient $\vec{AC} = 3\vec{AB}$

les vecteurs $\vec{AB}$ et $\vec{AC}$ étant colinéaires, les points A, B, C sont alignés et le point C appartient à la droite (AB)

Exercice 3

$$\left.\begin{array}{l} z_{\vec{AB}} = z_B - z_A = (2-3{,}9i) - (1{,}3+4{,}5i) = 0{,}7-8{,}4i \\ z_{\vec{DC}} = z_C - z_D = (1{,}2+3{,}1i) - (0{,}5+11{,}5i) = 0{,}7-8{,}4i \end{array}\right\}$$

donc $\vec{AB} = \vec{DC}$ donc ABCD est un parallélogramme

Exercice 4

$$a = 3\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

$$c = 5\sqrt{3}\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)$$

$$d = 4\left(\cos\left(\frac{-\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$$

$$b = 10\left(\cos\pi + i\sin\pi\right)$$

Exercice 5

1) $z_1 = \sqrt{2}\left[\cos\left(\frac{-\pi}{4}\right) + i\sin\left(\frac{-\pi}{4}\right)\right]$

$z_2 = \sqrt{2}\left[\cos\left(\frac{-\pi}{6}\right) + i\sin\left(\frac{-\pi}{6}\right)\right]$

$$\frac{z_1}{z_2} = \cos\left(\frac{-\pi}{12}\right) + \sin\left(\frac{-\pi}{12}\right) i \qquad (A)$$

2) $$\frac{z_1}{z_2} = \frac{2(1-i)}{\sqrt{6} - i\sqrt{2}} = \frac{2(1-i)(\sqrt{6}+i\sqrt{2})}{(\sqrt{6}-i\sqrt{2})(\sqrt{6}+i\sqrt{2})} = \frac{(1-i)(\sqrt{6}+i\sqrt{2})}{4}$$

$$\frac{z_1}{z_2} = \frac{(\sqrt{6}+\sqrt{2}) + i(-\sqrt{6}+\sqrt{2})}{4} \qquad (B)$$

3) En identifiant les formules (A) et (B) on obtient :

$$\cos\left(\frac{-\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\sin\left(\frac{-\pi}{12}\right) = \frac{-\sqrt{6}+\sqrt{2}}{4}$$