

Exercice 1

$$1) \quad (S) : \begin{cases} C(4) = 63 \\ C(10) = 165 \\ C(20) = 415 \end{cases}$$

$$2) \quad (S) \Leftrightarrow \begin{cases} 16a + 4b + c = 63 \\ 100a + 10b + c = 165 \\ 400a + 20b + c = 415 \end{cases}$$

$$(S) \Leftrightarrow \begin{pmatrix} 16 & 4 & 1 \\ 100 & 10 & 1 \\ 400 & 20 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 63 \\ 165 \\ 415 \end{pmatrix}$$

$$(S) \Leftrightarrow A \cdot X = B$$

$$A = \begin{pmatrix} 16 & 4 & 1 \\ 100 & 10 & 1 \\ 400 & 20 & 1 \end{pmatrix} \quad \text{et} \quad B = \begin{pmatrix} 63 \\ 165 \\ 415 \end{pmatrix}$$

$$\det(A) \neq 0$$

donc A est inversible

$$(S) \Leftrightarrow A^{-1} A X = A^{-1} B$$

$$\Leftrightarrow I_3 X = A^{-1} B$$

$$\Leftrightarrow X = A^{-1} B$$

$$X = \begin{pmatrix} 1/2 \\ 10 \\ 15 \end{pmatrix}$$

3) On obtient donc

$$C(x) = 0,15x^2 + 10x + 15$$

$$4) \quad C(30) = 765$$

Le coût de fabrication de 30 000 objets est de 765 000 €.

Exercice 2

$$z_1 = \frac{2}{29} - \frac{5}{29}i$$

$$z_3 = -27 + 31i$$

$$z_2 = -\frac{5}{169} + \frac{12}{169}i$$

$$z_4 = -1 + \frac{1}{2}i$$

Exercice 3

$$(3-2i)z = 3i+5$$

$$S = \left\{ \frac{9}{13} + \frac{19}{13}i \right\}$$

$$(2+i)z + 5 - 3i = 3z + 1 + i$$

$$S = \{4\}$$

Exercice 4

$$\begin{cases} 3z - z' = 6i \\ z + z' = 4 + 2i \end{cases}$$

$$S = \{(1+2i; 3)\}$$

Exercice 5

$$\begin{cases} f(0) = 0 \\ f(3) = -2,25 \\ f'(3) = 0 \end{cases} \Leftrightarrow \begin{cases} c = 0 \\ 9a + 3b + c = -2,25 \\ 6a + b = 0 \end{cases} \Leftrightarrow \begin{cases} c = 0 \\ 9a + 3b = -2,25 \\ 6a + b = 0 \end{cases}$$

$$\begin{pmatrix} 9 & 3 \\ 6 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -2,25 \\ 0 \end{pmatrix} \Leftrightarrow AX = B$$

A est inversible car $\det A = -9 \neq 0$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 9 & 3 \\ 6 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -2,25 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0,25 \\ -1,5 \end{pmatrix}$$

$$\boxed{f(x) = 0,25x^2 - 1,5x}$$