Exercice 1

a) $\Delta = -4$ $z_1 = -\frac{3}{2} - \frac{1}{2}i$ $\overline{z_1} = -\frac{3}{2} + \frac{1}{2}i$

b) $z^2 = -8$ $z = i\sqrt{8}$ ou $z = -i\sqrt{8}$ $S = \{-2i\sqrt{2} ; 2i\sqrt{2}\}$

c) $(z+2)(z+1) = z-3$

$z^2 + 2z + 5 = 0$

$\Delta = -16$ $z = -1-2i$ $\overline{z} = -1+2i$

$$S = \{-1-2i ; -1+2i\}$$

d) Changement de variable, on pose $Y = z^2$

$3Y^2 + 9Y - 30 = 0$

$\Delta = 441 = (21)^2$

$Y = -5$ ou $Y = 2$

$z^2 = -5$ $z^2 = 2$

$z = i\sqrt{5}$ ou $z = -i\sqrt{5}$ $z = \sqrt{2}$ ou $z = -\sqrt{2}$

$$S = \{-i\sqrt{5} ; i\sqrt{5} ; -\sqrt{2} ; \sqrt{2}\}$$

Exercice 2

$$\begin{cases} 2z - 3z' = 2-5i \\ z + 2z' = 1+i \end{cases} \Leftrightarrow \begin{cases} 2z - 3z' = 2-5i \\ -2z - 4z' = -2-2i \end{cases} \Leftrightarrow \begin{cases} z = 1+i-2z' \\ -7z' = -7i \end{cases}$$

$$\Leftrightarrow \begin{cases} z = 1+i-2i \\ z' = i \end{cases}$$

$$S = \{(1-i ; i)\}$$

Exercice 3

1) $|z| = \sqrt{16+48} = \sqrt{64} = 8$ $\qquad z = 8\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$

$$z = 8e^{-i\frac{\pi}{3}}$$

2) $|y| = \sqrt{3+3} = \sqrt{6}$

$$y = \sqrt{6}\, e^{\frac{3i\pi}{4}}$$

3) $|x| = \sqrt{\frac{225}{4} + \frac{75}{4}} = \sqrt{75} = 5\sqrt{3}$

$$x = 5\sqrt{3}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$$

$$x = 5\sqrt{3}\, e^{\frac{5i\pi}{6}}$$

Exercice 4

1) $P(-5) = (-5)^3 + 8\times 25 + 21(-5) + 30$

$P(-5) = 0$

-5 est une racine de P donc le polynôme se factorise par $(x+5)$

2) $P(z) = (z+5)(az^2 + bz + c)$

$P(z) = az^3 + (5a+b)z^2 + (5b+c)z + 5c$

En identifiant les coefficients, on obtient

$$\begin{cases} a = 1 \\ 5a+b = 8 \\ 5b+c = 21 \\ 5c = 30 \end{cases} \iff \begin{cases} a = 1 \\ b = 3 \\ c = 6 \end{cases}$$

$$P(z) = (z+5)(z^2 + 3z + 6)$$

3) $P(z) = 0 \iff z + 5 = 0$ ou $z^2 + 3z + 6 = 0$

$z = -5$ $\qquad \Delta = -15$

$z_1 = \frac{-3}{2} - \frac{\sqrt{15}}{2}i \qquad \overline{z_1} = \frac{-3}{2} + \frac{\sqrt{15}}{2}i$

$$S = \left\{-5\ ;\ -\frac{3}{2} - \frac{\sqrt{15}}{2}i\ ;\ -\frac{3}{2} + \frac{\sqrt{15}}{2}i\right\}$$

Exercice 5

1) $$Y = \frac{(\sqrt{2}+i\sqrt{6})(2+2i)}{(2-2i)(2+2i)} = \frac{2\sqrt{2}-2\sqrt{6}+i(2\sqrt{2}+2\sqrt{6})}{8} = \frac{\sqrt{2}-\sqrt{6}}{4} + i\,\frac{\sqrt{2}+\sqrt{6}}{4}$$

2) $$|z_1| = \sqrt{2+6} = 2\sqrt{2}$$

$$z_1 = 2\sqrt{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 2\sqrt{2}\, e^{i\frac{\pi}{3}}$$

$$|z_2| = 2\sqrt{2} = 2\sqrt{} \qquad z_2 = 2\sqrt{2}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$$

$$z_2 = 2\sqrt{2}\, e^{-i\frac{\pi}{4}}$$

3) $$Y = \frac{2\sqrt{2}\, e^{i\frac{\pi}{3}}}{2\sqrt{2}\, e^{-i\frac{\pi}{4}}} = e^{i\left(\frac{\pi}{3}+\frac{\pi}{4}\right)} = e^{\frac{7i\pi}{12}}$$

$$Y = e^{\frac{7i\pi}{12}} = \left[\cos\left(\frac{7\pi}{12}\right) + i\sin\left(\frac{7\pi}{12}\right)\right] \quad (*)$$

$$Y = \frac{\sqrt{2}-\sqrt{6}}{4} + i\,\frac{\sqrt{2}+\sqrt{6}}{4} \quad (**)$$

4) En identifiant (*) et (**) on obtient

$$\cos\frac{7\pi}{12} = \frac{\sqrt{2}-\sqrt{6}}{4}$$

$$\sin\frac{7\pi}{12} = \frac{\sqrt{2}+\sqrt{6}}{4}$$