Exercice 1

1) Solution particulière constante

$$x \to -\frac{5}{3}$$

2) Solutions de $y' = 3y$

$$x \to Ce^{3x}$$

Finalement les solutions s'écrivent

$$f(x) = Ce^{3x} - \frac{5}{3} \quad \text{avec } C \in \mathbb{R}$$

Exercice 2

Les solutions de l'équation différentielle sont de la forme

$$x \to Ce^{-\frac{1}{2}x}$$

$f(0) = 2 \iff Ce^0 = 2 \iff C = 2$ donc $f(x) = 2e^{-1/2\,x}$

De même, on obtient : $g(x) = e^{-1/2\,x}$

$j(x) = -e^{-1/2\,x}$ $\quad h(x) = -3e^{-1/2\,x}$

Exercice 3

1) Soit $P(x) = ax^2 + bx + c$ avec $(a,b,c) \in \mathbb{R}^* \times \mathbb{R} \times \mathbb{R}$

P solution de (E) $\iff 3(2ax + b) + 2(ax^2 + bx + c) = -x^2 + 2x - 5$

$$\iff (2a)x^2 + (6a + 2b)x + (3b + 2c) = -x^2 + 2x - 5$$

$$\iff \begin{cases} 2a = -1 \\ 6a + 2b = 2 \\ 3b + 2c = -5 \end{cases} \iff \begin{cases} a = -\frac{1}{2} \\ b = \frac{5}{2} \\ c = -\frac{25}{4} \end{cases}$$

$$P(x) = -\frac{1}{2}x^2 + \frac{5}{2}x - \frac{25}{4}$$

2) $S = \left\{ x \to Ce^{-2/3\,x} - \frac{1}{2}x^2 + \frac{5}{2}x - \frac{25}{4} \right\}$ avec $C \in \mathbb{R}$

Exercice 4

$$A = \left[\frac{t^3}{3} + t\right]_{t=0}^{t=2} = \frac{8}{3} + 2 = \frac{14}{3}$$

$$B = \left[-2\sqrt{t}\right]_{t=1}^{t=25} = -2(5-1) = -8$$

$$C = \left[-\frac{1}{5}\cos\left(5x + \frac{\pi}{2}\right)\right]_{x=0}^{x=\frac{\pi}{2}} = -\frac{1}{5}\cos(3\pi) = \frac{1}{5}$$

$$D = \int_1^2 (3t^{-4} + 2t^{-3})\,dt = \left[\frac{3t^{-3}}{-3} + \frac{2t^{-2}}{-2}\right]_1^2 = \left[-\frac{1}{t^3} - \frac{1}{t^2}\right]_{t=1}^{t=2}$$

$$= -\frac{1}{8} - \frac{1}{4} + 2 = \frac{13}{8}$$

$$E = \left[\frac{e^{3t}}{3}\right]_{t=0}^{t=1} = \frac{(e^3-1)}{3}$$

$$F = \int_e^{e^2} \Big(5 \times \underbrace{\frac{1}{t}}_{u'} \times \underbrace{\ln t}_{u}\Big) = \left[\frac{5}{2}(\ln t)^2\right]_{t=e}^{t=e^2} = \frac{5}{2}\left[(\ln(e^2))^2 - (\ln e)^2\right]$$

$$= \frac{5}{2}(4-1) = \frac{15}{2}$$

Exercice 5

$u(x) = 2x + 5$ $\qquad$ $v(x) = e^x$

$u'(x) = 2$ $\qquad$ $v'(x) = e^x$

$$I = \int_{-1}^{1} \underset{u}{(2x+5)} \underset{v'}{e^x}\, dx = \left[\underset{u}{(2x+5)} \underset{v}{e^x}\right]_{x=-1}^{x=1} - \int_{-1}^{1} 2e^x\, dx$$

$$= \left[(2x+5)e^x\right]_{-1}^{1} - \left[2e^x\right]_{-1}^{1}$$

$$= \left[(2x+3)e^x\right]_{x=-1}^{x=1}$$

$$= 5e^1 - e^{-1}$$

$$= \frac{5e^2 - 1}{e} \quad u.a.$$

Exercice 6

Soit m la valeur moyenne

$$m = \frac{1}{5-2} \int_2^5 \frac{2x}{(x^2+4)^2}\, dx$$

$$m = \frac{1}{3} \int_{-2}^{5} 2x\,(x^2+4)^{-2}\, dx$$

du type $u'u^n$
avec $u(x) = x^2 + 4$
$n = -2$

$$m = \frac{1}{3} \left[-\frac{1}{(x^2+4)} \right]_{x=-2}^{x=5}$$

$$m = -\frac{1}{3} \left(\frac{1}{29} - \frac{1}{8} \right)$$

$$m = \frac{7}{232}$$

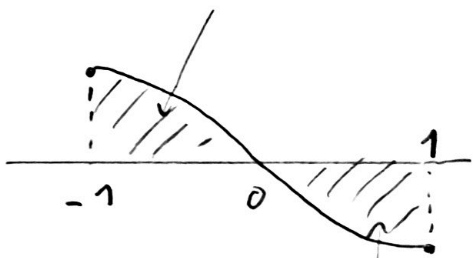
Exercice 7

1) Etude du signe de $f(x)$ sur $\mathbb{R}$

x	$-\infty$		-3		0		4		$+\infty$
x		$-$		$-$	0	$+$		$+$	
x^2-x-12		$+$	0	$-$		$-$	0	$+$	
$f(x)$		$-$	0	$+$	0	$-$	0	$+$	

2) Soit A l'aire cherchée

$\int_{-1}^{0} f > 0$



$\int_{0}^{1} f < 0$

$$A = \int_{-1}^{1} |f(x)|\,dx$$

$$A = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} -f(x)\,dx$$

$$A = \int_{-1}^{0} f(x)\,dx + \int_{1}^{0} f(x)\,dx$$

$$A = \int_{-1}^{0} (x^3-x^2-12x)\,dx + \int_{1}^{0} (x^3-x^2-12x)\,dx$$

$$A = \left[\frac{x^4}{4} - \frac{x^3}{3} - 6x^2\right]_{-1}^{0} + \left[\frac{x^4}{4} - \frac{x^3}{3} - 6x^2\right]_{x=1}^{x=0}$$

$$A = -\left(\frac{1}{4} + \frac{1}{3} - 6\right) - \left(\frac{1}{4} - \frac{1}{3} - 6\right)$$

$$A = 12 - \frac{1}{4} - \frac{1}{3} - \frac{1}{4} + \frac{1}{3}$$

$$A = 11{,}5 \text{ u.a.}$$