Exercice 1

1) $f(-7) = -2$ $\quad f'(-7) = -3$ $\quad f(-4) = 5$ $\quad f'(-4) = 0$

$f(3) = 4$ $\quad f'(3) = \frac{1}{2}$

2) Tangente au point d'abscisse 0

$y = f'(0)x + f(0)$

$y = 2x - 1$

3) Tangente au point d'abscisse 7

$y = f'(7)(x-7) + f(7)$

$y = -\frac{1}{5}(x-7) + 2$

$y = -\frac{1}{5}x + \frac{17}{5}$

Exercice 2

1)

a) $\dfrac{f(1+h) - f(1)}{h} = \dfrac{\frac{1}{1+h} - \frac{1}{1}}{h} = \dfrac{1}{h} \times \dfrac{1-(1+h)}{1+h}$

$\dfrac{f(1+h) - f(1)}{h} = \dfrac{-h}{h(1+h)} = -\dfrac{1}{1+h}$

b) $\lim\limits_{h \to 0} \dfrac{f(1+h) - f(1)}{h} = \lim\limits_{h \to 0} -\dfrac{1}{1+h} = -1$

f est dérivable en 1 et on a $f'(1) = -1$

2) $\dfrac{g(-2+h) - g(-2)}{h} = \dfrac{-(-2+h)^2 + 3(-2+h) - 1 - (-11)}{h}$

$= \dfrac{-h^2 + 7h}{h}$

$= -h + 7$

$\lim\limits_{h \to 0} \dfrac{g(-2+h) - g(h)}{h} = 7$ donc $g'(-2) = 7$

Exercice 3

) $1 + 3 + S_1 = 1 + 3 + 3^2 + \cdots + 3^{13}$

$$4 + S_1 = \frac{1 - 3^{14}}{1 - 3}$$

$$S_1 = 2\,391\,480$$

) $S_2 = -5 + 1 + 7 + \cdots + 67$ $\qquad 67 = -5 + 6 \times 12$

$S_2 = 67 + 61 + \cdots - 5$

$2 S_2 = 62 \times 13$

$S_2 = 31 \times 13$

$S_2 = 403$

) $S_3 = 1 + (-3) + (-3)^2 + (-3)^3 + \cdots + (-3)^{11}$

$$S_3 = \frac{1 - (-3)^{12}}{1 - (-3)}$$

$S_3 = -132\,860$

Exercice 4

$u_0 = 15\,000$

$u_1 = 15\,000 \times 1{,}04$

$\vdots$

$u_n = 15000 \times 1{,}04^n$

On cherche n tel que

$15\,000 \times 1{,}04^n \geq 45\,000$

$\Leftrightarrow \quad 1{,}04^n \geq 3$

$n \geq 29$

Pour tripler un capital placé à 4%, il faut 29 années.

Exercice 5

1) $u_1 = 0,6 \quad u_2 = -0,48$

2)
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for k in range(1,8):
    U = (U - 3)/5
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Exercice 6

1) $u_2 = 500 \times 1,02 = 510$
$u_3 = 510 \times 1,02 = 520,2$

2) $u_{n+1} = 1,02\, u_n$
(u_n) est géométrique de raison 1,02 et de premier terme $u_1 = 500$

3) $\forall n \geq 1 \quad u_n = 500 \times 1,02^{n-1}$

$u_{20} = 500 \times 1,02^{19} \approx 728$ €

La 20ème année, il touchera 728 € de prime

4) $S = u_1 + u_2 + \dots + u_{20}$

$S = 500 + 500 \times 1,02 + \dots + 500 \times 1,02^{19}$

$S = 500\left(1 + 1,02 + 1,02^2 + \dots + 1,02^{19}\right)$

$S = 500 \times \dfrac{1 - 1,02^{20}}{1 - 1,02}$

$S \approx 12\,149$ €