Exercice 1

1) $V_1 = 1$

$V_2 = \frac{4}{3}$

2) $U_2 = 5$

$U_3 = 12$

Exercice 2

1) $u_1 = 10\,000 \times 0,97 = 9700$

$u_2 = 9700 \times 0,97 = 9409$

2) $u_{n+1} = 0,97\, u_n$

3) $u_n = 10\,000 \times 0,97^n$

4) On cherche n (le plus petit) tel que $10\,000 \times 0,97^n < 5000$ (On utilise la table de la calculatrice)

$0,97^n < 0,5$

$n \geqslant 23$

En Python :

```
U = 10 000
N = 0
while U > 5000 :
    U = U * 0,97
    N = N + 1
print(N)
```

Exercice 3

1) $u_1 = -3$

$u_2 = 9$

$u_3 = 4,2$

2) $u_1 - u_0 = -3$

$u_2 - u_1 = 12$

$u_1 - u_0 \neq u_2 - u_1$

donc (u_n) n'est pas arithmétique

$\frac{u_2}{u_1} = -3$

$\frac{u_3}{u_2} = \frac{4,2}{9}$

$\frac{u_2}{u_1} \neq \frac{u_3}{u_2}$ donc (u_n) n'est pas géométrique

Exercice 4

1) $u_5 = u_1 \times q^4 \iff q^4 = \frac{3072}{12} \iff q^4 = 256$

$\iff q = 4$

$u_7 = 3072 \times 4^2 = 49152$

2) $v_1 + v_2 + \dots + v_8 = 92$

$\iff -6 + (-6+r) + (-6+2r) + \dots (-6+7r) = 92$

$\iff -6 \times 8 + r + 2r + 3r + \dots + 7r = 92$

$\iff r(1+2+3+4+5+6+7) = 92 + 48$

$\iff 28r = 140$

$\iff r = 5$

$v_8 = v_1 + 7r$

$v_8 = -6 + 7 \times 5$

$v_8 = 29$

3) $u_{20} = u_{12} + 8r \iff 8r = 49 - 25 \iff r = 3$

On obtient $u_n = 25 + 3(n-12)$

$\iff \boxed{u_n = -11 + 3n}$ pour tout $n \geq 1$

$u_1 = -8$ $\quad u_{30} = 79$

$S = -8 - 5 - 2 + \dots + 70 + 73 + 79$

$S = \frac{-8+71}{2} \times 30$

$S = 71 \times 15$

$S = 1065$

Exercice 5

1) $S_1 = 2 + 3 + \dots + 48 + 49$

$S_1 = 49 + 48 + \dots + 3 + 2$

$2S_1 = 51 \times 48$

$S_1 = 1224$

2) $S_2 = 3 + 11 + \dots + 115 + 123$

$S_2 = 123 + 115 + \dots + 11 + 3$

$2S_2 = 126 \times 16$

$S_2 = 1008$

3) $S_3 = 2 + 4 + \dots + 4096$

$S_3 = 2 + 2^2 + \dots + 2^{12} = \frac{1-2^{13}}{1-2} - 1$

$S_3 = \frac{1-8192}{-1} - 1$

$S_3 = 8191 - 1$

$S_3 = 8190$

4) $S_4 = 1 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \dots + \left(\frac{1}{2}\right)^8 = \frac{1-\left(\frac{1}{2}\right)^9}{1-\frac{1}{2}} = 2 \times \left(1 - \frac{1}{512}\right)$

$S_4 = 2 \times \frac{511}{512}$

$S_4 = \frac{511}{256}$

Exercice 6

1)  6 tuyaux sont empilés

2) $n + n-1 + n-2 + \dots + 2 + 1 = \frac{n(n+1)}{2}$

3) $\frac{n(n+1)}{2} = 351$

$\Leftrightarrow n^2 + n - 702 = 0$

$\Leftrightarrow n = -27$ ou $n = 26$

$S = \{26\}$

Il y a 26 tuyaux sur le sol et il y a 26 étages.

Exercice bonus

$$\begin{cases} a+b+c = 120 \\ abc = 59160 \end{cases} \Leftrightarrow \begin{cases} (b-r) + b + (b+r) = 120 \\ abc = 59160 \end{cases}$$

$$\Leftrightarrow \begin{cases} 3b = 120 \\ abc = 59160 \end{cases} \Leftrightarrow \begin{cases} b = 40 \\ (40-r) \times 40 \times (40+r) = 59160 \end{cases}$$

$$\Leftrightarrow \begin{cases} b = 40 \\ (40-r)(40+r) = 1479 \end{cases} \Leftrightarrow \begin{cases} b = 40 \\ 1600 - r^2 = 1479 \end{cases}$$

$$\begin{cases} b = 40 \\ r^2 - 121 = 0 \end{cases} \Leftrightarrow \begin{cases} b = 40 \\ r = -11 \text{ ou } r = 11 \end{cases}$$

$$\Leftrightarrow \begin{cases} a = 29 \\ b = 40 \\ c = 51 \end{cases}$$

Les trois termes consécutifs de la suite arithmétique sont :

29 ; 40 ; 51